Calculus AB

4-3

Definite Integrals

Definition of a Definite Integral -

if f is defined on the closed interval [a,b] and the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

exists, then f is integreble on [a,b] and the limit is denoted by

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \int_{a}^{b} f(x)\,dx$$

The limit is called the definite integral of f from a to b, where a is the lower limit of integration, and b is the upper limit of integration.

Evaluate the definite integral by the limit definition. (pg 272)

3) $\int_{4}^{10} 6\,dx = \lim_{n\to\infty} \overset{\Delta x}{\frac{6}{n}} \sum_{i=1}^{n} \overset{c}{6} = \frac{6}{n}\left(\overset{c\cdot n}{6\cdot n}\right) = 36$

Evaluate the definite integral by the limit definition. $x_i = -1 + i\Delta x$

$-1 + \frac{2i}{n}$

5) $\int_{-1}^{1} x^3\, dx = \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(-1 + \frac{2i}{n}\right)^3$ $\quad -1 \quad 3\left(\frac{2i}{n}\right) - 3\left(\frac{2i}{n}\right)^2 \; 1\left(\frac{2i}{n}\right)^3$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} -1\frac{n^3}{n^3} + \frac{6i \cdot n^2}{n \cdot n^2} \; \frac{12i^2 \cdot n}{n^2 \cdot n} + \frac{8i^3}{n^3}$$

$$= \lim_{n\to} \frac{2}{n^4} \sum_{i=1}^{n} -n^3 + 6in^2 - 12i^2 n + 8i^3$$

$$= \frac{2}{n^4}\left(-n^3 \cdot n + \frac{6n^2 \cdot n(n+1)}{2} - \frac{12n \cdot n(n+1)(2n+1)}{6} + \frac{8n^2(n+1)^2}{4}\right)$$